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Inductive Logic

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The current state of inductive logic is puzzling. Survey presentations are recurrently offered (see, e.g., [20, 30, 52], classic textbooks by leading scholars are reedited and new ones are published (see [28, 59], respectively) and a very rich and extensive handbook was entirely dedicated to the topic just a few years ago [23]. Among the contributions to this very volume, however, one finds forceful arguments to the effect that inductive logic is not needed and that the belief in its existence is itself a misguided illusion ([50]; also see [51] for a consonant line of argument), while other distinguished observers have eventually come to see at least the label as "slightly antiquated" ([43], p. 291).

What seems not to have lost any of its currency is the *problem* which inductive logic is meant to address. Inference from limited ascertained information to uncertain hypotheses is ubiquitous in learning, prediction and discovery. The logical insight that such kind of inference is fallible may well be a platitude after Hume, but its real-life counterparts remain painfully prominent nonetheless – missed medical diagnoses [66] or judicial errors [46] illustrate effectively. And the otherwise amazing success of inference under uncertainty in scientific inquiry as well as in many everyday matters is still a live issue in the study of human knowledge, cognition and behavior [63].

Having sketched out this bewildering background, I will not try to tame it in any way – I think it'd be unwise. Instead, I will settle on one specific way to pursue inductive logic which, although popular and somehow traditional, is far from uncontroversial. This view (i) crucially involves the analysis of how given premises

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or data affect the credibility of conclusions or hypotheses of interest and (ii) relies on probability to represent rational degrees of credence. In short, I will assume that investigating inductive logic essentially amounts to pursuing (some variant of) Bayesian confirmation theory. To be sure, a good deal of current philosophical work in related areas involves reasoned criticism of either point (i) (see, e.g., [1, 4]), point (ii) (see, e.g., [61]), or both (see, e.g., [39, 45, 47]. Perhaps, by presenting a coherent selection of recent achievements, this concise overview will persuade some skeptic that inductive-logical research through confirmation theory stands as a deserving endeavour. But that's more in hope than in purpose – I'd be more than content with providing a clear target, be that for criticism, or rather for agreement and development.

1 Basics

A probabilistic theory of inductive confirmation can be spelled out by the definition of a function $C_P(h, e)$ representing the degree of support that hypothesis h receives from evidence e (relative to probability function P). The machinery needed is relatively standard. One can take some basic logical language $\boldsymbol{U}$ (finite, for simplicity), the subset $\boldsymbol{U}_C$ of its consistent formulae and the set $\boldsymbol{P}$ of all regular probability functions defined over $\boldsymbol{U}$. (This set up is known to be very convenient but not entirely innocent. Festa [17] and Kuipers [42], 44 ff., discuss some limiting cases that are left aside thereby.) Confirmation will thus be represented by $C_P(h, e) : \{\boldsymbol{U}_C \times \boldsymbol{U}_C \times \boldsymbol{P}\} \to \Re$ and will have relevant probability values as its building blocks, according to the following fundamental postulate.

(F) *Formality.*
There exists a function g such that, for any $h, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e) = g[P(h \wedge e), P(h), P(e)]$.

The probability distribution over the algebra generated by h and e is entirely determined by $P(h \wedge e)$, $P(h)$ and $P(e)$. Hence, (F) simply states that $C_P(h, e)$ depends on that distribution, and nothing else. (The label for this assumption is taken from Tentori et al. [64, 65], where formality is shown not to hold unrestrictedly for the human mind.) In a probabilistic theory of inductive confirmation, the principle below is also usually taken to hold by default (at least, I'm not aware of any explicit contention). It states that, for any fixed hypothesis h, the final probability and confirmation always move in the same direction in the light of data.

(P) *Final probability.*
For any $h, e_1, e_2 \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e_1) \gtreqless C_P(h, e_2)$ if and only if $P(h|e_1) \gtreqless P(h|e_2)$.

Importantly, (P) is a comparative (ordinal) principle. In fact, the ordinal level of analysis is a solid middleground between a purely qualitative and a thoroughly quantitative (metric) notion of confirmation. In particular, ordinal notions are sufficient to move "upwards" to the qualitative level, according to the following definition:

(QC) *Qualitative confirmation* (*from ordinal relations*).
For any $h, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, e confirms / is neutral for / disconfirms h if and only if $C_P(h, e) \gtreqless C_P(\neg h, e)$.

As we will see in a moment, for the qualitative notion of confirmation to get its proper meaning, we only need to add the following (where "$\top$" denotes a tautology):

(T) *Tautological evidence*.
For any $h_1, h_2 \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, $C_P(h_1, \top) = C_P(h_2, \top)$.

(T) implies that any hypothesis is equally "confirmed" by empty evidence, as it were. For the present purposes, I suggest to call $C_P(h, e)$ a *probabilistic relevance measure of* (*inductive*) *confirmation* if and only if, beyond the technical requirement (F), it satisfies both (P) and (T). The following simple result motivates this terminology.

Theorem 1 *Given definition (QC), (P) and (T) imply that, for any $h, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, e confirms / is neutral for / disconfirms h if and only if $P(h|e) \gtreqless P(h)$.*

Proof For any $h, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$:

$P(h|e) \gtreqless P(h)$
if and only if $P(\neg h) \gtreqless P(\neg h|e)$ (probability calculus)
if and only if $P(h|e) \gtreqless P(h|\top)$ and $P(\neg h|\top) \gtreqless P(\neg h|e)$ (probability calculus)
if and only if $C_P(h, e) \gtreqless C_P(h, \top)$ and $C_P(\neg h, \top) \gtreqless C_P(\neg h, e)$ (P)
if and only if $C_P(h, e) \gtreqless C_P(h, \top) = C_P(\neg h, \top) \gtreqless C_P(\neg h, e)$ (T)
if and only if e confirms / is neutral for / disconfirms h. (QC)

□

Theorem 1, of course, clarifies the core idea of inductive confirmation as probabilistic relevance, namely, that the credibility of a hypothesis can be *changed* in either a positive (confirmation in a strict sense) or negative way (disconfirmation) by the evidence concerned. As surprising as it may seem, I think it is fair to say that full understanding of these very basic features of inductive confirmation as relevance has been achieved only in relatively recent times. (To my mind, the foregoing is a particularly polished characterization drawing on insights to be found in [2, 3, 14, 17, 48]. Also see [7].)

2 A Fragment of Axiomatics

It is now well understood that the quantitative analysis of probabilistic confirmation (as well as of other cognate notions, see, e.g., [9]) can be characterized axiomatically. Consider, in particular, the following principles (see [13] for relevant references).

(A1) *Disjunction of alternative hypotheses*.
For any $h_1, h_2, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, if $P(h_1 \wedge h_2) = 0$, then $C_P(h_1 \vee h_2, e) \gtreqless C_P(h_1, e)$ if and only if $P(h_2|e) \gtreqless P(h_2)$.

(A2) *Law of likelihood*.
For any $h_1, h_2, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, $C_P(h_1, e) \gtreqless C_P(h_2, e)$ if and only if $P(e|h_1) \gtreqless P(e|h_2)$.

(A3) *Modularity* (*for conditionally independent data*).
For any $h, e_1, e_2 \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, if $P(e_1| \pm h \wedge e_2) = P(e_1| \pm h)$, then $C_P(h, e_1|e_2) = C_P(h, e_1)$.

Interestingly, one can rely on these as characteristic axioms to neatly sort out the notion of inductive confirmation into three classical and ordinally divergent families of measures, as follows.

Theorem 2 [12, 13, 31]

Let $C_P(h, e)$ be a probabilistic relevance measure of confirmation. Then:

(i) (A1) holds if and only if $C_P(h, e)$ is a *probability difference measure*, that is, if there exists a strictly increasing function f such that, for any $h, e \in U_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e) = f[D_P(h, e)]$, where $D_P(h, e) = P(h|e) - P(h)$;

(ii) (A2) holds if and only if $C_P(h, e)$ is a *probability ratio measure*, that is, if there exists a strictly increasing function f such that, for any $h, e \in U_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e) = f[R_P(h, e)]$, where $R_P(h, e) = P(h|e)/P(h)$;

(iii) (A3) holds if and only if $C_P(h, e)$ is a *likelihood ratio measure*, that is, if there exists a strictly increasing function f such that, for any $h, e \in U_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e) = f[L_P(h, e)]$, where $L_P(h, e) = P(e|h)/P(e|\neg h)$.

As a matter of theoretical understanding, it seems to me that Theorem 2 represents an interesting achievement in that it carves out the distinctive traits of the most popular Bayesian models of inductive confirmation at the ordinal level. (For useful lists including somewhat less popular options and further discussion, one can see, e.g., [3, 18, 24, 27, 54, 56].)

The ordinal notion of confirmation is arguably of greater theoretical relevance than its metrical counterpart, because ordinal divergences, unlike purely quantitative differences, imply opposite comparative judgments for some evidence-hypothesis pairs of significant philosophical interest (see [19] for a now classical discussion). A refinement from the ordinal to a properly metrical level can still be of value, however, and surely much convenient for tractability and applications. A quantitative requirement that is sometimes put forward is the following form of additivity (see [26, 48]):

(*SA*) *Strict additivity.*
For any $h, e_1, e_2 \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e_1 \wedge e_2) = C_P(h, e_1) + C_P(h, e_2|e_1)$.

If a strictly additive behavior is imposed, one functional form is singled out for the quantitative representation of confirmation corresponding to each of the clauses of Theorem 2 above, that is:

(i) $D_P(h, e) = P(h|e) - P(h)$;

(ii) $R_P(h, e) = \log\left[\frac{P(h|e)}{P(h)}\right]$;

(iii) $L_P(h, e) = \log\left[\frac{P(e|h)}{P(e|\neg h)}\right]$.

(The bases of the logarithms are of course assumed to be greater than 1.)

Recently, arguments have been offered by Huber [37] and Milne [48] in favor of D, and by Glass and McCartney [25] and Park [53] in favor of L. As for R, it can be seen as conveying key tenets of the so-called "likelihoodist" position about evidential reasoning, as suggested by Fitelson [22] (see [55] for a classical statement of likelihoodism, and [6, 60] for consonant arguments and inclinations). Some have seen different measures as possibly capturing "distinct, complementary notions of evidential support" (Hájek and Joyce [29], p. 123, describing Joyce's [38] position; also see Schlosshauer and Wheeler [57] and Steel [62] for tempered forms of pluralism), but this same plurality has prompted other scholars to be skeptical or dismissive of the prospects for a quantitative theory of confirmation. According to Howson [35], for instance, "there are few transparently essential properties that degree of confirmation should have" (p. 184; also see Kyburg and Teng [44], 98 ff.). Less so, I would like to submit, if one takes the connection between confirmation theory and inductive logic seriously enough. This will be the focus of the following section.

3 What Does it Take to Generalize Deductive Logic?

It is a long-standing idea, going back to Carnap at least, that confirmation theory should be analogous to classical deductive logic in some substantial sense, thus providing a theory of partial entailment and partial refutation. This is of course a most credible project precisely if one is willing to align or even identify the very ideas of confirmation theory and inductive logic. Hawthorne [30] seems sympathetic with this aspiration, but pessimistic in diagnosis, pointing out that "an inductive logic must, it seems, deviate from this paradigm" (see the discussion in Crupi and Tentori [10]). Let us have a closer look.

Presumably, it was this deductive / inductive analogy that inspired Hempel's [33] early proposal of his "entailment condition" as a requirement for confirmation theory.

(EC) *Entailment condition* (*qualitative*).
For any $h, e \in \boldsymbol{U}_C$ with h contingent and any $P \in \boldsymbol{P}$, if $e \models h$ then e confirms h

As is well known, Hempel's qualitative theory of confirmation fulfils (EC) and thus embeds logical entailment as a special case in a straightforward way (see [7]). Within a quantitative theory of confirmation, we can aim at something more, such as the following:

(EC*) *Entailment condition* (*ordinal extension*).
For any $h, e_1, e_2, e_3 \in \boldsymbol{U}_C$ with h contingent and any $P \in \boldsymbol{P}$ if $e_1 \models h$, $e_2 \models h$ and $e_3 \not\models h$, then $C_P(h, e_1) = C_P(h, e_2) > C_P(h, e_3)$.

According to (EC*), not only is classical entailment retained as a case of confirmation, it also represents a limiting case: it is the strongest possible form of confirmation that a fixed hypothesis h can receive. Happily enough, all confirmation measures above – D, R, and L – do imply (EC*) (see [14], p. 79). So far so good.

Hempel also took the following as obviously true (indeed, by definition; see [34], p. 127):

(CC) *Confirmation complementarity* (*qualitative*).
For any $h, e \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, e confirms h if and only if e disconfirms $\neg h$

Here, confirmation and disconfirmation are meant to be interdefinable via negation, just like logical entailment and refutation. What about the ordinal extension of this?

(CC*) *Confirmation complementarity* (*ordinal extension*).
$C_P(\neg h, e)$ is a strictly decreasing function of $C_P(h, e)$, that is, for any $h_1, h_2, e_1, e_2 \in \boldsymbol{U}_C$ and any $P \in \boldsymbol{P}$, $C_P(h_1, e_1) \gtreqless C_P(h_2, e_2)$ if and only if $C_P(\neg h_1, e_1) \lesseqgtr C_P(\neg h_2, e_2)$.

(CC*) neatly reflects Keynes' remark that "an argument is always as near to proving or disproving a proposition, as it is to disproving or proving its contradictory" ([41], p. 80). Indeed, a long list of scholars have endorsed (CC*), including Carnap ([5], § 67), Kemeny and Oppenheim ([40], p. 309), and Eells and Fitelson ([16], p. 134). Measures D and L instantiate (CC*) in the most simple and elegant way, that is, $C(h, e) = -C(\neg h, e)$. Yet R *fails* this condition ([14], p. 86), a fact described as no less than "damning" by Milne [48]. Apparently, R is thus ruled out as a sound confirmation-theoretic generalization of deductive logic.

As pointed out by Fitelson ([21], p. 502), the Carnapian approach to inductive logic was also driven by the following principle, similar to (EC*) above, but more stringent:

(L) *Logicality*.
For any $h_1, h_2, h_3, e_1, e_2, e_3 \in \boldsymbol{U}_C$ with h_1, h_2 and h_3 contingent and any $P \in \boldsymbol{P}$:

(i) if $e_1 \models h_1$, $e_2 \models h_2$ and $e_3 \not\models h_3$, then $C_P(h_1, e_1) = C_P(h_2, e_2) > C_P(h_3, e_3)$;
(ii) if $e_1 \models \neg h_1$, $e_2 \models \neg h_2$ and $e_3 \not\models \neg h_3$, then $C_P(h_1, e_1) = C_P(h_2, e_2) < C_P(h_3, e_3)$.

According to the logicality condition, entailment (refutation) is the one strongest possible form of confirmation (disconfirmation) even across distinct hypotheses. As Fitelson ([21], p. 506) observes, logicality singles out L, being violated by D (as well as by R).

End of the story? Well, there's one more little *coup de théâtre*, with which I would like to conclude this detour and wrap up some implications from this as well as the previous section. The deductive-logical notions of entailment and refutation (contradiction) also exhibit the following well-known properties:

Contraposition of entailment. Entailment is contrapositive, but not commutative. That is, it holds that e entails h ($e \models h$) if and only if $\neg h$ entails $\neg e$ ($\neg h \models \neg e$), while it does not hold that e entails h if and only if h entails e ($h \models e$).

Commutativity of refutation. Refutation, on the contrary, is commutative, but not contrapositive. That is, it holds that e refutes h ($e \models \neg h$) if and only if h refutes e ($h \models \neg e$), while it does not hold that e refutes h if and only if $\neg h$ refutes $\neg e$ ($\neg h \models \neg\neg e$).

The confirmation-theoretic counterparts would seem pretty straightforward:

(A4) *Contraposition of confirmation.*
For any $h, e \in \boldsymbol{U_C}$ and any $P \in \boldsymbol{P}$, if e confirms h, then $C_P(h, e) = C_P(\neg e, \neg h)$.

(A5) *Commutativity of disconfirmation.*
For any $h, e \in \boldsymbol{U_C}$ and any $P \in \boldsymbol{P}$, if e disconfirms h, then $C_P(h, e) = C_P(e, h)$.

There is only one way, it turns out, to embed (A4) and (A5) within a probabilistic approach to confirmation, as is shown by the following result:

Theorem 3 [10]

Let $C_P(h, e)$ be a probabilistic relevance measure of confirmation. Then (A4) and (A5) hold if and only if $C_P(h, e)$ is a *relative distance measure*, that is, if there exists a strictly increasing function f such that, for any $h, e \in U_C$ and any $P \in \boldsymbol{P}$, $C_P(h, e) = f[Z_P(h, e)]$, where:

$$Z_P(h, e) = \begin{cases} \frac{P(h|e)-P(h)}{1-P(h)} & \text{if } P(h|e) \geqslant P(h) \\ \frac{P(h|e)-P(h)}{P(h)} & \text{if } P(h|e) < P(h) \end{cases}$$

What is most intriguing is that with a relative distance measure like Z one gets all the rest for free, namely, (*EC**), (*CC**), and (L) all follow. (And more besides: see [8, 11, 15]; also see [10], where the *relative distance* label, originally due to [36], is explained.)

Relative distance measures had independent historical background in early research in automated expert reasoning [32, 58], and got their share of criticism as an explication of evidential support (see [3, 25, 49]). What we're left with here, anyway, and my final suggestion, is simply as follows. *Suppose* that one sees inductive logic as best addressed within some probabilistic theory of confirmation *and* takes it seriously that inductive logic should generalize deductive-logical relationships. *Then* a significant set of compelling constraints arise, contrary to concerns that are sometimes raised (e.g., [35]). Moreover, and contrary to worries of opposite sign (e.g., [30]), such constraints can all be satisfied.

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References

1. Achinstein, P. (Ed.) (2005). *Scientific evidence: philosophical theories and applications*. Baltimore: John Hopkins University Press.
2. Atkinson, D. (2012). Confirmation and justification: a commentary on Shogenji's measure. *Synthese*, *184*, 49–61.
3. Brössel, P. (2013). The problem of measure sensitivity redux. *Philosophy of Science*, *80*, 378–397.
4. Brössel, P., & Huber, F. (2014). Bayesian confirmation: a means with no end. *British Journal for the Philosophy of Science*. doi:10.1093/bjps/axu004.
5. Carnap, R. (1950/1962). *Logical foundations of probability*. Chicago: University of Chicago Press.
6. Chandler, J. (2013). Contrastive confirmation: some competing accounts. *Synthese*, *190*, 129–138.
7. Crupi, V. (2014). Confirmation. In E.N. Zalta (Ed.) *The stanford encyclopedia of philosophy*. (Fall 2014 Edition). http://plato.stanford.edu/archives/fall2014/entries/confirmation/.
8. Crupi, V., & Tentori, K. (2010). Irrelevant conjunction: statement and solution of a new paradox. *Philosophy of Science*, *77*, 1–13.
9. Crupi, V., & Tentori, K. (2012). A second look at the logic of explanatory power (with two novel representation theorems). *Philosophy of Science*, *79*, 365–385.
10. Crupi, V., & Tentori, K. (2013). Confirmation as partial entailment: a representation theorem in inductive logic. *Journal of Applied Logic*, *11*, 364–372. [Erratum in *Journal of Applied Logic*, *12*, 230–231].
11. Crupi, V., & Tentori, K. (2014). State of the field: measuring information and confirmation. *Studies in the History an Philosophy of Science A*, *47*, 81–90.
12. Crupi, V., & Tentori, K. (forthcoming). Confirmation theory. In A. Hájek & C. Hitchcock (Eds.) *Oxford handbook of philosophy and probability*. Oxford: Oxford University Press.
13. Crupi, V., Chater, N., & Tentori, K. (2013). New axioms for probability and likelihood ratio measures. *British Journal for the Philosophy of Science*, *64*, 189–204.
14. Crupi, V., Festa, R., & Buttasi, C. (2010). Towards a grammar of Bayesian confirmation. In M. Suárez, M. Dorato, M. Rédei (Eds.) *Epistemology and methodology of science* (pp. 73–93). Dordrecht: Springer.
15. Crupi, V., Tentori, K., & Gonzalez, M. (2007). On Bayesian measures of evidential support: theoretical and empirical issues. *Philosophy of Science*, *74*, 229–52.
16. Eells, E., & Fitelson, B. (2002). Symmetries and asymmetries in evidential support. *Philosophical Studies*, *107*, 129–42.
17. Festa, R. (1999). Bayesian confirmation. In M. Galavotti & A. Pagnini (Eds.) *Experience, reality, and scientific explanation* (pp. 55–87). Dordrecht: Kluwer.
18. Festa, R. (2012). For unto every one that hath shall be given. Matthew properties for incremental confirmation. *Synthese*, *184*, 89–100.
19. Fitelson, B. (1999). The plurality of Bayesian measures of confirmation and the problem of measure sensitivity. *Philosophy of Science*, *66*, S362–S378.
20. Fitelson, B. (2005). Inductive logic. In S. Sarkar & J. Pfeifer (Eds.) *Philosophy of science. An encyclopedia* (pp. 384–393). New York: Routledge.
21. Fitelson, B. (2006). Logical foundations of evidential support. *Philosophy of Science*, *73*, 500–512.
22. Fitelson, B. (2007). Likelihoodism, Bayesianism, and relational confirmation. *Synthese*, *156*, 473–89.
23. Gabbay, D., Hartmann, S., & Woods, J. (2011). *Handbook of the history of logic, Vol. 10: Inductive logic*. Amsterdam: Elsevier.
24. Glass, D.H. (2013). Confirmation measures of association rule interestingness. *Knowledge-Based Systems*, *44*, 65–77.
25. Glass, D.H., & McCartney, M. (2014). A new argument for the likelihood ratio measure of confirmation. *Acta Analytica*. doi:10.1007/s12136-014-0228-6.
26. Good, I.J. (1984). The best explicatum for weight of evidence. *Journal of Statistical Computation and Simulation*, *4*, 294–299.
27. Greco, S., Slowinski, R., & Szczech, I. (2012). Properties of rule interestingness measures and alternative approaches to normalization of measures. *Information Sciences*, *216*, 1–16.
28. Hacking, I. (2001). *An introduction to probability and inductive logic*. Cambridge: Cambridge University Press.

29. Hájek, A., & Joyce, J. (2008). Confirmation. In S. Psillos & M. Curd (Eds.) *Routledge companion to the philosophy of science* (pp. 115–129). New York: Routledge.
30. Hawthorne, J. (2012). Inductive logic. In E.N. Zalta (Ed.) *The stanford encyclopedia of philosophy*, (Winter 2012 Edition). http://plato.stanford.edu/archives/win2012/entries/logic-inductive/.
31. Heckerman, D. (1988). An axiomatic framework for belief updates. In J.F. Lemmer & L.N. Kanal (Eds.) *Uncertainty in artificial intelligence* (pp. 11–22), 2. Amsterdam: North-Holland.
32. Heckerman, D., & Shortliffe, E.H. (1992). From certainty factors to belief networks. *Artificial Intelligence in Medicine*, *4*, 35–52.
33. Hempel, C.G. (1945). Studies in the logic of confirmation. *Mind*, *54*, 1–26, 97–121.
34. Hempel, G.G. (1943). A purely syntactical definition of confirmation. *Journal of Symbolic Logic*, *8*, 122–143.
35. Howson, C. (2000). *Hume's problem: induction and the justification of belief*. Oxford: Oxford University Press.
36. Huber, F. (2007). Confirmation and induction. *Internet Encyclopedia of Philosophy*. http://www.iep.utm.edu/conf-ind/#SH6b.
37. Huber, F. (2008). Assessing theories, Bayes style. *Synthese*, *161*, 89–118.
38. Joyce, J. (1999). *The foundations of causal decision theory*. Cambridge: Cambridge University Press.
39. Kelly, K., & Glymour, C. (2004). Why probability does not capture the logic of scientific justification. In C. Hitchcock (Ed.), *Contemporary debates in the philosophy of science* (pp. 94–114). London: Blackwell.
40. Kemeny, J., & Oppenheim, P. (1952). Degrees of factual support. *Philosophy of Science*, *19*, 307–324.
41. Keynes, J. (1921). *A treatise on probability*. London: Macmillan.
42. Kuipers, T. (2000). *From instrumentalism to constructive realism*. Dordrecht: Reidel.
43. Kyburg Jr., H.E. (2008). Inductive logic and inductive reasoning. In J.E. Adler, & L.J. Rips (Eds.) *Reasoning* (pp. 291–301). New York: Cambridge University Press.
44. Kyburg, H.E., & Teng, C.M. (2001). *Uncertain inference*. New York: Cambridge University Press.
45. Levi, I. (2010). Probability logic, logical probability, and inductive support. *Synthese*, *172*, 97–118.
46. Liebman, J., Fagan, J., West, V., & Lloyd, J. (2000). Capital attrition: error rates in capital cases, 1973–1995. *Texas Law Review*, *78*, 1839–1865.
47. Mayo, D. (2003). Severe testing as a guide for inductive learning. In: H.E. Kyburg Jr. & M. Thalos (Eds.), *Probability is the very guide of life* (pp. 89–117). Chicago: Open Court.
48. Milne, P. (2012). On measures of confirmation. manuscript.
49. Milne, P. (2014). Information, confirmation, and conditionals. *Journal of Applied Logic*, *12*, 252–262.
50. Musgrave, A. (2011). Popper and hypothetico-deductivism. In D. Gabbay, S. Hartmann, J. Woods, (Eds.) *Handbook of the history of logic, Vol. 10: Inductive logic* (pp. 205–234). Amsterdam: Elsevier.
51. Norton, J.D. (2010). There are no universal rules for induction. *Philosophy of Science*, *77*, 765–777.
52. Paris, J. (2011). Pure inductive logic. In L. Horsten & R. Pettigrew (Eds.) *The continuum companion to philosophical logic* (pp. 428–449). London: Continuum.
53. Park, I. (2014). Confirmation measures and collaborative belief updating. *Synthese*, *191*, 3955–3975.
54. Roche, W., & Shogenji, T. (2014). Dwindling confirmation. *Philosophy of Science*, *81*, 114–137.
55. Royall, R. (1997). *Statistical evidence: a likelihood paradigm*. London: Chapman & Hall.
56. Schippers, M. (2014). Probabilistic measures of coherence: from adequacy constraints towards pluralism. *Synthese*, *191*, 3821–3845.
57. Schlosshauer, M., & Wheeler, G. (2011). Focussed correlation, confirmation, and the jigsaw puzzle of variable evidence. *Philosophy of Science*, *78*, 376–392.
58. Shortliffe, E.H., & Buchanan, B.G. (1975). A model of inexact reasoning in medicine. *Mathematical Biosciences*, *23*, 351–379.
59. Skyrms, B. (2000). *Choice and chance: an introduction to inductive logic*. Belmont: Wadsworth Thomson Learning.
60. Sober, E. (1990). Contrastive empiricism. In C.W. Savage (Ed.) *Minnesota studies in the philosophy of science, Vol. 14: Scientific theories* (pp. 392–412). Minneapolis: University of Minnesota Press.
61. Spohn, W. (2012). *The laws of belief*. Oxford: Oxford University Press.
62. Steel, D. (2007). Bayesian Confirmation theory and the likelihood principle. *Synthese*, *156*, 55–77.
63. Tenenbaum, J., Kemp, C., Griffiths, T., & Goodman, N. (2011). How to grow a mind: statistics, structure, and abstraction. *Science*, *331*, 1279–1285.

64. Tentori, K., Crupi, V., & Osherson, D. (2007). Determinants of confirmation. *Psychonomic Bulletin & Review*, *14*, 877–83.
65. Tentori, K., Crupi, V., & Osherson, D. (2010). Second-order probability affects hypothesis confirmation. *Psychonomic Bulletin & Review*, *17*, 129–34.
66. Winters, B., Custer, J., Galvagno Jr., S.M., Colantuoni, E., Kapoor, S.G., Lee, H., Goode, V., Robinson, K., Nakhasi, A., Pronovost, P., & Newman-Toker, D. (2012). Diagnostic errors in the intensive care unit: a systematic review of autopsy studies. *BMJ Quality and Safety*, *21*, 894–902.