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State of the field: Measuring information and confirmation

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ABSTRACT

The aim of this paper is to survey and discuss some key connections between information and confirmation within a broadly Bayesian framework. We mean to show that treating information and confirmation in a unified fashion is an intuitive and fruitful approach, fostering insights and prospects in the analysis of a variety of related notions such as belief change, partial entailment, entropy, the value of experiments, and more besides. To this end, we recapitulate established theoretical achievements, disclose a number of underlying links, and provide a few novel results.

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1. Introduction

Information is a ubiquitous term, occurring across philosophy and the sciences with a great variation in meaning. *Confirmation*, on the other hand, is a term of art in contemporary philosophy of science, defined as the impact of evidence on hypotheses. What do these two notions have in common? They both are, it turns out, central concepts when one deals with rational inference and inquiry. Loosely speaking, it seems clear that the impact of a piece of evidence (data, premise) on a given hypothesis (theory, conclusion) must reflect how the former affects an antecedent state of information concerning the latter. Relatedly, a rational agent would gather evidence because it provides information concerning certain possible states of affairs, i.e., for it can confirm/disconfirm relevant hypotheses.

The aim of this paper is to survey and discuss some key connections between information and confirmation within a broadly Bayesian framework. Indeed, a common view about information is that it is inversely related to probability (an assumption which Floridi, 2013 calls "inverse relationship principle" after Barwise, 1997, p. 491). So getting to know that the outcome of a draw from a well-shuffled deck happens not to be the seven of clubs is not very informative, for that was quite likely to be the case to begin with, and one's epistemic state would be altered only to a limited extent by this discovery. Being told that the outcome of the draw is a picture of hearts provides more information in comparison, because this singles out a small subset of possibilities that was initially rather improbable. In philosophy, this basic idea found its canonical formulation in seminal work by Bar-Hillel and Carnap (1953), who famously discussed two distinct formal representations of the information conveyed by a statement s:

$$inf_R(s) = \log[1/P(s)]$$

$$inf_D(s) = 1 - P(s)$$

The base of the logarithm is taken to be greater than unity (in the following, we will always comply with the use of $\log_2$, a fairly common choice). For the moment, subscripts "R" and "D" simply reflect the *ratio* and the *difference* involved in the corresponding expressions; but this notation will gain more relevance further on in our discussion. These classical analyses have not remained unchallenged (see, e.g., Floridi, 2004; also see Cevolani, 2013 for a neat and recent discussion), but stand as a sound basis at least for our purposes.

Mathematically, inf_R is also pivotal to so-called information theory, a well-established discipline founded by Claude Shannon (1948) with major implications in engineering and other applied

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sciences (see McKay, 2003). Ever since Bar-Hillel (1955), warnings have been flagged that conceptual confusion can potentially arise from this overlap of formalisms. In fact, in standard information-theoretic applications, $P(s)$ does *not* represent the credibility of a statement in an epistemic context, but rather the relative frequency of occurrence of a symbol in a code—a crucial difference of interpretation. However, inf_R ended up having rather wide currency in philosophy of science and related areas, too, and probably more than inf_D. A key point of this paper, anyway, is to keep both possibilities open. Thus, we will often write *inf* (with no subscript) to denote a measure of information that could be either of inf_R or inf_D.[1]

Let us now turn briefly to confirmation. A probabilistic theory of confirmation can be spelled out by a function representing the degree of support that hypothesis h receives from evidence e relative to some probability distribution P. Here, we'll rely on the background of a finite set of possible worlds, a corresponding Boolean language $\boldsymbol{L}$, and the set $\boldsymbol{P}$ of all regular probability functions that can be defined over the latter.[2] $\boldsymbol{L}_C$ will denote the subset of the consistent formulae in $\boldsymbol{L}$ (i.e., those denoting a non-empty set of possibilities). Confirmation will then be represented by some function $conf(h,e)$: $\{\boldsymbol{L}_C \times \boldsymbol{L}_C \times \boldsymbol{P}\} \rightarrow \Re$ and will have relevant probability values as its building blocks (a feature named *formality* in Tentori, Crupi, & Osherson, 2007, 2010).[3]

Note that, if an appropriate function $conf(h,e)$ is identified, a qualitative notion of confirmation can be easily derived, as follows:

(Q) *Qualitative confirmation*. A hypothesis is confirmed by some evidence just in case its complement is disconfirmed. Formally: for any $h,e \in \boldsymbol{L}_C$ (with h non-tautological) and any $P \in \boldsymbol{P}$, e confirms/is neutral for/disconfirms h if and only if $conf(h,e) \gtreqless conf(\neg h,e)$.

As for $conf(h,e)$ itself, two basic requirements will suffice for our present purposes:

(F) *Final probability*. For any fixed hypothesis, final (posterior) probability and confirmation always move in the same direction in the light of data. Formally: for any $h,e,f \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, $conf(h,e) \gtreqless conf(h,f)$ if and only if $P(h|e) \gtreqless P(h|f)$.

(T) *Tautological evidence*. Any hypothesis is equally "confirmed" by empty (tautological) evidence. Formally: for any $h,k \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, $conf(h,\top) = conf(k,\top)$.

We will call $conf(h,e)$ a *measure of confirmation* if and only if it satisfies both (F) and (T). As a motivation for this choice, let us first note that (F) is a virtually unchallenged principle in probabilistic theories of confirmation (see Crupi, Chater, & Tentori, 2013 for a list of references). Moreover, coupling (F) and (T) is sufficient to imply, via definition (Q), the traditional notion that, for any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, e confirms/is neutral for/disconfirms h if and only if $P(h|e) \gtreqless P(h)$.[4]

Relying on the basic points above, we now mean to present a collection of results and open issues concerning how information and confirmation are connected. Some parts of this contribution will thus draw on a background of well-known theoretical achievements, but we will often dig out material that we think is scattered or currently underappreciated in the philosophical literature, and sometimes interpolate entirely novel elements. In the next section, we will first address inference, where some given evidence and a single target hypothesis are at issue. Further on, we will be concerned more with search and inquiry, that is, with the expected value of collecting potentially relevant evidence. A final section will then outline some implications and prospects for future investigation.

2. Information, confirmation, and the impact of evidence

2.1. From information to confirmation as belief change

Being a decreasing function of $P(s)$, $inf(s)$ reflects the "unexpectedness" of s. If the evidence acquired decreases (increases) the degree of unexpectedness of a hypothesis of interest, the credibility of such hypothesis is thereby positively (negatively) affected. A simple way to convey this natural idea is to represent the *belief change* concerning h provided by e, $bc(h,e)$, by means of the plain difference between $inf(h)$ and $inf(h|e)$ (see Milne, 2014). Notably, two classical confirmation measures are thus immediately recovered[5]:

$$bc_R(h,e) = inf_R(h) - inf_R(h|e) = \log[P(h|e)/P(h)]$$

$$bc_D(h,e) = inf_D(h) - inf_D(h|e) = P(h|e) - P(h)$$

For both measures, it readily follows that $bc(h,h) = inf(h)$, an implication strongly welcomed by Milne (2012) (also see Huber, 2008 in this respect). Moreover, both measures exhibit an appealing additive behavior, in that $bc(h,e \wedge f) = inf(h)$

[1] Many authors, following Bar-Hillel and Carnap, do retain both inf_R and inf_D, either as useful theoretical constructs (e.g., Hintikka, 1968, 1970; Hintikka & Pietarinen, 1966; Kuipers, 2006; Milne, 2014; Pietarinen, 1970) or as a target of criticism (e.g., Maher, 1993, pp. 234 ff.; Levi, 1967, p. 374). When a choice is made, however, inf_R quite often prevails (see, for instance, Cox, 1961, Ch. 2; Mura, 2006, p. 196; van Rooij 2009, p. 170; Törnebohm, 1964, Ch. 3), although inf_D is found in classical works such as Hempel and Oppenheim (1948, p. 171) and Popper (1959, p. 387), and appears as a central notion—*uncertainty*—in Adams's probability logic (see, for instance, Adams, 1998, p. 31). Howson and Franklin (1985) are particularly firm in their preference for inf_R against inf_D, which they criticize as involved in Popper and Miller's (1983) famous attack on inductive probability. However, our reliance on inf_D throughout this paper does not imply in any way Popper and Miller's controversial assumption that $e \rightarrow h$ represents all of the content of h that goes beyond e (see Redhead, 1985). Indeed, logically, one can safely retain inf_D and still reject Popper and Miller's argument as unsound. (These clarifications were prompted by useful comments by an anonymous reviewer, which we gratefully acknowledge.)

[2] This set up is known to be very convenient, but has limitations. Festa (1999) and Kuipers (2000, pp. 44 ff.) discuss some important cases that are left aside here.

[3] Properly speaking, the notation should also indicate that C depends on some P in $\boldsymbol{P}$. One should write, for instance, $conf_P(h,e)$, or $conf(h,e,P)$. This amount of rigor would burden subsequent parts of our discussion inconveniently, though.

[4] *Proof*. For any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$:

$$\begin{array}{lll}
P(h|e) \gtreqless P(h) & \text{if and only if } P(\neg h) \gtreqless P(\neg h|e) & \text{(probability calculus)} \\
& \text{if and only if } P(h|e) \gtreqless P(h|\top) \text{ and } P(\neg h|\top) \gtreqless P(\neg h|e) & \text{(probability calculus)} \\
& \text{if and only if } C_P(h,e) \gtreqless C_P(h,\top) \text{ and } C_P(\neg h,\top) \gtreqless C_P(\neg h,e) & \text{(by } F) \\
& \text{if and only if } C_P(h,e) \gtreqless C_P(h,\top) = C_P(\neg h,\top) \gtreqless C_P(\neg h,e) & \text{(by } T) \\
& \text{if and only if } e \text{ confirms/is neutral for/disconfirms } h. & \text{(by } Q)
\end{array}$$

Of course, we are dealing with the idea of confirmation as *relevance* here ("increase of firmness", in Carnap's, 1950/62 terminology: also see Good, 1968, p. 134; Salmon, 1975). In a view of confirmation as "firmness", on the other hand, $conf(h,e)$ would simply amount to an increasing function of $P(h|e)$. Interestingly, one can characterize *this* notion by replacing (T) with the following condition of *local equivalence*: for any $h,k,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, if h and k are made logically equivalent by e, then $conf(h,e) = conf(k,e)$ (see Crupi, 2013; Crupi & Tentori, in press; Schippers, 2013).

[5] Measures ordinally equivalent to $bc_R(h,e)$ have been discussed in epistemology ever since Keynes (1921, pp. 165 ff.), while $bc_D(h,e)$ was influentially put forward by Carnap (1950/62, p. 361).

$- inf(h|e) + inf(h|e) - inf(h|e \wedge f) = bc(h,e) + bc(h,f|e)$. As we will see shortly, important differences exist between $bc_R(h,e)$ and $bc_D(h,e)$, however (see Fig. 1 for an illustration). Up to ordinal equivalence, they can be characterized, respectively, by the following specific axioms:

(LL) *Law of likelihood*. For any $h,k,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, $bc(h,e) \gtreqless bc(k,e)$ if and only if $P(e|h) \gtreqless P(e|k)$.

(DH) Disjunction *of alternative hypotheses*. For any $h,k,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, if $P(h \wedge k) = 0$, then $bc(h,e) \gtreqless bc(h \vee k,e)$ if and only if $P(k|e) \gtreqless P(k)$.

Apart from the uniqueness result concerning the ordinal (comparative) behavior of $bc_D(h,e)$ (a proof is in Crupi & Tentori, in press), condition (DH) was hardly ever mentioned in the literature. (LL), on the contrary, has been recurrently discussed and counts overt supporters (see Crupi et al., 2013 for various references). Indeed, as it implies (LL), $bc_R(h,e)$ appears to convey a key tenet of so-called "likelihoodist" position about evidence (see Royall, 1997 for a classical statement, Chandler, 2013; Sober, 1990 for consonant arguments, and Fitelson, 2007; Steel, 2007 for discussions). It is then interesting to note that a number of arguments have piled up recently which clearly seem to favor $bc_D(h,e)$ over $bc_R(h,e)$. We will now present three of them.

(i) A serious problem with (LL) is that, along with basic principle (F) above, it implies *commutativity*, so that for any confirmation measure satisfying (LL) it turns out that $bc(h,e) = bc(e,h)$. Eells and Fitelson (2002) showed through elementary examples how unsettling this is. For instance, consider a draw from a well-shuffled deck, with e = "the card drawn is hearts" and h = "the card drawn is red". We then have the unsound equality $bc_R(h,e) = bc_R(e,h)$, even though e is conclusive evidence for h, while h is not conclusive evidence for e. The difference measure gets this case right, instead, with the obvious ranking $bc_D(h,e) = {}^1\!/_2 > {}^1\!/_4 = bc_D(e,h)$.

(ii) According to basic principle (Q) above, e confirms h if and only if it disconfirms $\neg h$ (Hempel, 1943, p. 127, already saw this as a plain definitional truth). Measure $bc_D(h,e)$ yields a simple and elegant quantitative extension of this principle, which many have found appealing (see Carnap, 1950/1962, § 67; Eells & Fitelson, 2002, p. 134; Kemeny & Oppenheim, 1952, p. 309), namely:

(C) *Complementarity*. For any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, $bc(h,e) = -bc(\neg h,e)$.

Crupi, Festa, and Buttasi (2010, p. 86) proved, on the other hand, that not only is (LL) inconsistent with (C), but it even contradicts the underlying idea that the greater the increase in credibility that h receives from some evidence e, the greater the decrease that its negation $\neg h$ suffers from e. The latter requirement is a purely ordinal counterpart of (C) and seems unobjectionable. Yet it is violated by measures satisfying (LL), like $bc_R(h,e)$, a problem described as no less than "damning" by Milne (2012).

(iii) From now on, we will call a *query* a (finite) set of mutually exclusive and jointly exhaustive statements in $\boldsymbol{L}_C$. So, for a query $Q = \{q_1, \ldots, q_n\}$, $q_i \in \boldsymbol{L}_C$ for each i ($1 \leq i \leq n$), $P(q_j \wedge q_m) = 0$ if $j \neq m$ ($1 \leq j,m \leq n$), and $\sum_{q_i \in Q} P(q_i) = 1$. Moreover, assuming $P \in \boldsymbol{P}$, $P(q_i) > 0$ for any $q_i \in Q$ (because P is regular). Every query thus defined implies a corresponding partition of the set of possibilities. The "empty" query $\mathcal{T} = \{\top\}$ is included as inducing the limiting case of partitions, i.e., the set of all possible worlds.[6] Now consider a single

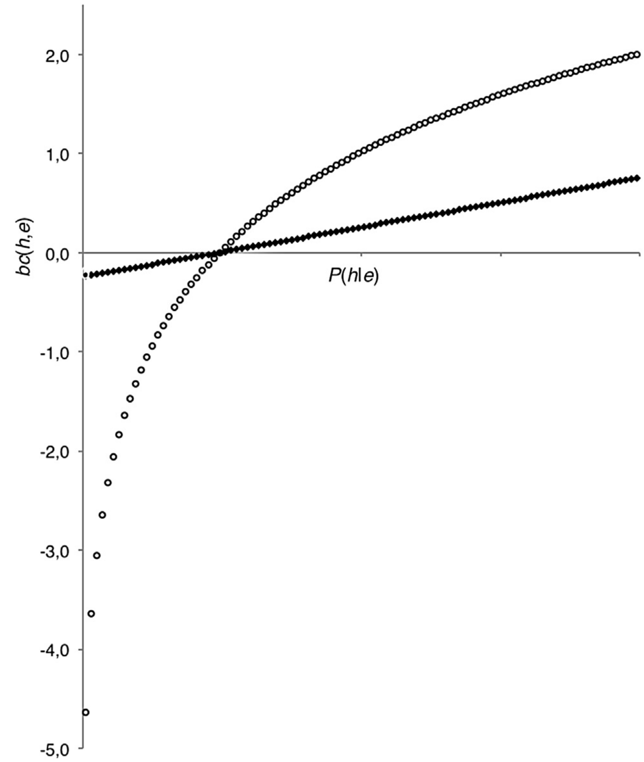

Fig. 1. A graphical illustration of bc_R (○) and bc_D (◆) as distinct measures of belief change as a function of posterior probability, $P(h|e)$. Prior probability $P(h)$ is fixed at 0.25. Accordingly, zero change in belief obtains for both measures precisely when $P(h|e) = 0.25 = P(h)$.

hypothesis h and a query $E \in \boldsymbol{Q}$ (with $\boldsymbol{Q}$ the set of all queries). Here is a neat point recently made by Milne (2012). In a standard Bayesian framework, the prior degree of belief in h just is the expected value of the posterior across all elements in E, because $P(h) = \sum_{e_i \in E} P(h|e_i)P(e_i)$. A natural interpretation is that the degree of belief in h is stable as long as the current state of uncertainty concerning E endures (and no other belief change occurs). Accordingly, Milne argues, the change in belief about h should be null in expectation, for otherwise one would apparently have grounds to change one's degree of belief in h in advance of obtaining any evidence, contrary to the assumption of stability. If one accepts this line of argument, then the appeal of $bc_D(h,e)$ against $bc_R(h,e)$ is further increased, for only the former satisfies the relevant adequacy condition that, for any $h \in \boldsymbol{L}_C$, any $E \in \boldsymbol{Q}$ and any $P \in \boldsymbol{P}$, $\sum_{e_i \in E} bc(h,e_i)P(e_i) = 0$, while $bc_R(h,e)$ violates it.

So far, we considered one straightforward connection between information and confirmation, pointing out how two popular confirmation measures emerge depending on whether inf_R or inf_D is presupposed. Beyond some appealing shared features, we argued, convergent arguments exist by which bc_D seems to fare better than bc_R as an explication of change in belief. But pluralists of either more radical or moderate strands would allow for "distinct, complementary notions of evidential support" to be analyzed in different ways (Hájek & Joyce, 2008, p. 123). In particular, there may well be other concepts of confirmation or support with a prominent connection with information. As we mean to show now, the idea of *partial entailment* (and *partial refutation*) is a case in point.

[6] The mathematical structure of queries and partitions is more extensively explored, for instance, in Domotor (1970, pp. 173 ff.), Niiniluoto (1976, pp. 265 ff.), and van Rooij (2009).

2.2. From information to confirmation as partial entailment

Consider the role that information (or *content*, in some formulations) often plays in the interpretation of deductive–logical relationships. Classical entailment preserves truth by banning novelty, as it were. So we are used to say, at least informally, that a conclusion h follows from (a conjunction of) premise(s) e when there is no information left in h that is not already conveyed (if implicitly) by e. That is, once e is assumed, h does not really say anything more.[7] This immediately suggests an analysis of partial entailment as a measure of the proportion of the initial amount of information of the conclusion h that is removed once e is assumed as a premise. In symbols, we are talking about the proportion of $inf(h)$ that is spanned by $inf(h) - inf(h|e)$. This idea applies, of course, when assuming e does reduce the initial amount of information in h—so that $inf(h|e) < inf(h)$—, thereby providing some positive support at all. Otherwise, namely when $inf(h|e) \geq inf(h)$, a treatment of *partial refutation* would be needed. And here, following Keynes (1921, p. 80), one would surely want "an argument [to be] always as near to proving or disproving a proposition, as it is to disproving or proving its contradictory". This compelling requirement motivates the above condition of *Complementarity* in a different context: the degree of partial entailment of h by e should correspond to a comparably severe degree of partial refutation of $\neg h$. The two basic constraints just mentioned, in turn, suffice to imply the following definition of partial entailment and refutation:

$$part(h,e) = \begin{cases} \dfrac{inf(h) - inf(h|e)}{inf(h)} & \text{if } inf(h|e) < inf(h) \\ -\dfrac{inf(\neg h) - inf(\neg h|e)}{inf(\neg h)} & \text{otherwise} \end{cases}$$

A plausible account of partial entailment is of course an old chestnut and aspiration in philosophical work on inductive logic, pursued by Keynes (1921) and Carnap (1950/62) and criticized by Salmon (1969), among others. Interestingly, depending on whether $inf_R(h,e)$ or $inf_D(h,e)$ is adopted, the analysis above leaves us with two more existing confirmation measures, for simple algebraic manipulations yield:

$$part_R(h,e) = \begin{cases} \dfrac{\log P(h|e) - \log P(h)}{-\log P(h)} & \text{if } P(h|e) > P(h) \\ \dfrac{\log P(\neg h) - \log P(\neg h|e)}{-\log P(\neg h)} & \text{otherwise} \end{cases}$$

$$part_D(h,e) = \begin{cases} \dfrac{P(h|e) - P(h)}{P(\neg h)} & \text{if } P(h|e) > P(h) \\ \dfrac{P(h|e) - P(h)}{P(h)} & \text{otherwise} \end{cases}$$

Mura (2006, 2008) proposed measure $part_R$ in a dense discussion of how the logical notions of content, independence, and entailment can be extended in a probabilistic framework, while Crupi, Tentori, and Gonzalez (2007) and Crupi and Tentori (2013) argued for $part_D$ as a sound explication of partial entailment and refutation.[8]

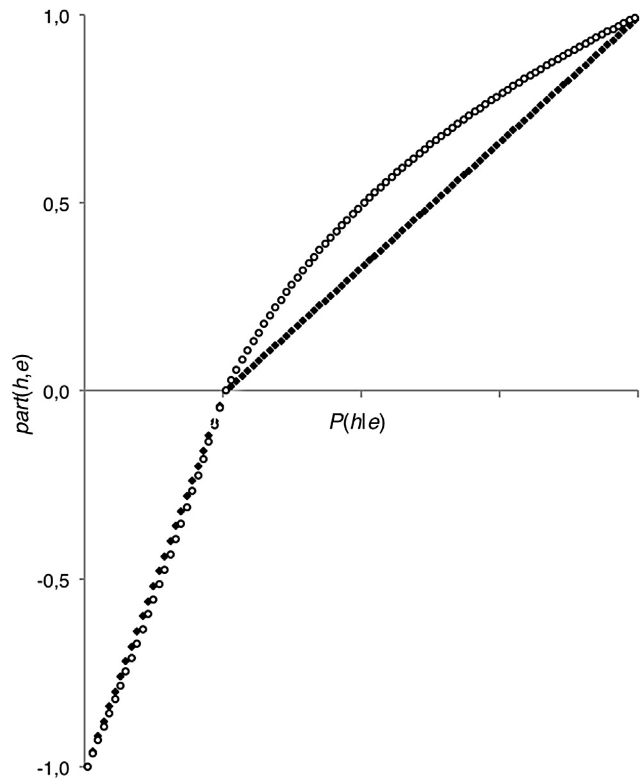

Fig. 2. A graphical illustration of $part_R$ (○) and $part_D$ (◆) as distinct measures of the degree of partial entailment of h from e as a function of posterior probability, $P(h|e)$. Prior probability $P(h)$ is fixed at 0.25. Accordingly, zero partial entailment obtains for both measures precisely when $P(h|e) = 0.25 = P(h)$.

As Fig. 2 shows, $part_R$ and $part_D$ may come very close in their numerical outcomes. This is largely because, for whatever value of $P(h)$, they both range between -1 and $+1$ and both satisfy *Complementarity* (by design, as explained above). Yet the structural properties that they exhibit have different implications of theoretical significance.

A result by Atkinson (2012) implies that the *positive branch* of $part_R$ is essentially characterized by the equivalence $part(h,e) = part(k,e) = part(h \wedge k,e)$ for hypotheses h and k that are probabilistically independent both unconditionally and conditional on e. This was called *special conjunction requirement* (*SCR*) and advocated as an adequacy condition by Shogenji (2012). However, the target explicandum of Shogenji's contribution was "epistemic justification", not partial entailment. So a certain tension arises here. Relying on (*SCR*) across the board, Shogenji devised an explication of epistemic justification formally identical to the

[7] Here again, worries did sometimes emerge concerning this received view: see D'Agostino (2013).

[8] Measure $part_D$ was also discussed in theoretical work on *certainty factors*, a central notion to represent uncertain reasoning in early expert systems (see Heckerman & Shortliffe, 1992; Shortliffe & Buchanan, 1975) and then more recently in data mining (see Glass, 2013; Greco, Slowinski, & Szczech, 2012) and cognitive psychology (Rusconi et al., 2014). Moreover, early appearances exist for the positive branches of either of the *part* measures. Niiniluoto and Tuomela (1973, p. 67) mention both in a discussion of the "informational power" of evidence. Moreover, the positive branch of $part_R$ was central to Törnebohm's (1964, 1966) analysis of evidential support, while the positive branch of $part_D$ is ordinally equivalent to Gaifman's (1979, p. 120) measure of confirmation, was then suggested by Rips (2001, p. 129, fn. 1) as a measure of the strength of inductive arguments, and has been included in recent work by Schlosshauer and Wheeler (2011, p. 381), and Wheeler and Scheines (2011, p. 40).

positive branch of $part_R$, which however can be shown to contradict Keynes's idea that $part(\neg h,e)$ should be a decreasing function of $part(h,e)$ (see Milne, 2012). On the other hand, Keynes's condition is secured if $part_R$ is taken as a whole, but then no argument has been put forward, to the best of our knowledge, for why (*SCR*) should hold for partial entailment but fail for partial refutation.

As for $part_D$, let us first note that a particularly neat connection can be drawn with the belief change counterpart bc_D. This can be informally framed by means of a nice analogy that we adapt from Nelson (2008, p. 149). Consider current degrees of belief as a location on a journey. A possible destination of such journey is the ascertainment (probability 1) of hypothesis h. Then $bc_D(h,e)$ is a straightforward metric of the distance traveled towards or away from that destination (positive vs. negative change in belief about h, respectively). On the other hand, $part_D(h,e)$ measures how much has been covered of the distance that *could* have been traveled either towards or away from that destination—the latter quantity being either $1 - P(h) = P(\neg h)$, or just $P(h)$, respectively. (In fact, $part_D$ was discussed much along similar lines in Crupi et al., 2010, p. 75; Crupi & Tentori, 2010, p. 7.)

One further remark conveys the distinctive appeal of $part_D$ as a quantitative extension of logical entailment. Entailment is contrapositive, but not commutative: it holds that e entails h ($e \vDash h$) if and only if $\neg h$ entails $\neg e$ ($\neg h \vDash \neg e$), while it does not hold that e entails h if and only if h entails e ($h \vDash e$). One might then consider favorably having $part(h,e) = part(\neg e,\neg h)$ for any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, but only as long as positive probabilistic relevance obtains between e and h. Refutation, on the contrary, is commutative, but not contrapositive: it holds that e refutes h ($e \vDash \neg h$) if and only if h refutes e ($h \vDash \neg e$), while it does not hold that e refutes h if and only if $\neg h$ refutes $\neg e$ ($\neg h \vDash \neg\neg e$). Accordingly, one might favorably consider having $part(h,e) = part(e,h)$ for any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, but only as long as *negative* probabilistic relevance obtains between e and h. As it happens, $part_D$ is the only confirmation measure satisfying these constraints, up to ordinal equivalence (see Crupi & Tentori, 2013).[9]

3. Information, confirmation, and the utility of experiments

3.1. Entropies and entropy reduction

Let us consider the expected value of inf_R over the possible outcomes of some query $H \in \boldsymbol{Q}$, that is:

$$ent_R(H) = \sum_{h_i \in H} inf_R(h_i)P(h_i)$$

The label *ent* is of course inspired by the notion of entropy in Shannon's (1948) framework. In fact, several authors have seen ent_R as a sound representation of the amount of uncertainty concerning a hypothesis set H as a whole, and therefore the expected informativeness of finding out what particular hypothesis in that set is true (see, e.g., Paris, 1994, pp. 76 ff.; van Rooij 2009, pp. 173 ff.; Rosenkrantz, 1977, pp. 11 ff.; Sneed, 1967). The counterpart expression for inf_D is simply:

$$ent_D(H) = \sum_{h_i \in H} P(h_i)inf_D(h_i)$$

which turns out to be equivalent to:

$$ent_D(H) = 1 - \sum_{h_i \in H} P(h_i)^2$$

In the form above, ent_D sometimes occurs under the label of *quadratic entropy* and is closely connected to so-called Gini–Simpson index of diversity, fairly common in biological applications and beyond (Gini, 1912; Rao, 1982; Simpson, 1949).

Measures ent_R and ent_D exhibit a number of theoretically intriguing connections: ent_R is known to have the same mathematical form of physical entropy in statistical thermodynamics (see Werndl & Frigg, 2011), while ent_D equals to 1 minus so-called *informational energy*, a quantity introduced by Onicescu (1966) as an analog to kinetic energy (see Domotor, 1970, p. 188). Furthermore, both ent_R and ent_D are special cases of a parametric family of functions discussed in physics as *Tsallis entropies* (after Tsallis, 1988), but already studied by Havrda and Charvát (1967). A simple form is the following:

$$ent^{\alpha}(H) = \frac{1}{\alpha - 1}\left(1 - \sum_{h_i \in H} P(h_i)^{\alpha}\right)$$

One immediately gets ent_D for $\alpha = 2$, while ent_R is recovered in the limit for $\alpha \to 1$. Various axiomatic approaches have been pursued in the definition of entropy measures (see Chakrabarti & Ghosh, 2009; Csizár, 2008 for recent overviews and further references). On the background of a few plausible and shared assumptions, ent_R and ent_D can be distinctively characterized by their respective additive behavior (see Abe, 2000). In particular, for independent queries, that is, for any $X,Y \in \boldsymbol{Q}$ and any $P \in \boldsymbol{P}$, such that $X \perp_P Y$ ("$\perp_P$" denotes independence relative to probability distribution P), ent_R turns out to yield a more straightforward formula:

$$ent_R(X{\ast}Y) = ent_R(X) + ent_R(Y)$$

$$ent_D(X{\ast}Y) = ent_D(X) + ent_D(Y) - ent_D(X)ent_D(Y)$$

Here, "$\ast$" denotes an operation of combination defined as follows: for any $X,Y \in \boldsymbol{Q}$, $X{\ast}Y = \{x \wedge y | x \in X \text{ and } y \in Y\}$ (it is easy to show that the set of queries $\boldsymbol{Q}$, as it was presented above, is closed under "$\ast$"). In squarely quantitative terms, however, ent_R and ent_D do not differ too much, apart from an inconsequential variance in range, as illustrated in Fig. 3.

In all our previous discussion of confirmation, we were concerned with how some piece of evidence e can change the credibility of one particular target hypothesis h. But clearly, when some relevant evidence is acquired, a change occurs from a prior to a posterior distribution over the *whole* set of hypotheses of interest H. Being defined over queries, the notion of entropy allows for an analysis of the extent to which the initial entropy concerning H is reduced (or increased) by a new item of evidence e. Two variations thus arise for the difference between the earlier and the later amount of entropy with regards to H as prompted by e, often called the *information gain*:

$$ig_R(H,e) = ent_R(H) - ent_R(H|e)$$

$$ig_D(H,e) = ent_D(H) - ent_D(H|e)$$

[9] Milne (2012) criticizes $part_D$ for failing a *generalized additivity* constraint, namely, that $part(h,e \wedge f)$ be always determined by $part(h,e)$ and $part(h,f|e)$ (on this sole basis, he does not even list $part_D$ as a genuine candidate measure of confirmation). The same criticism would immediately apply to $part_R$, too. However, for both measures, generalized additivity does hold as long as e and f are convergent in their impact (either both confirmatory or both disconfirmatory) and can only fail if they have opposite effects on the credibility of h. This may well be a consequence of the fact that two notions are being jointly explicated (partial entailment *and refutation*) which, although appropriately related (e.g., via *Complementarity*), do not (*should* not!) feature identical properties.

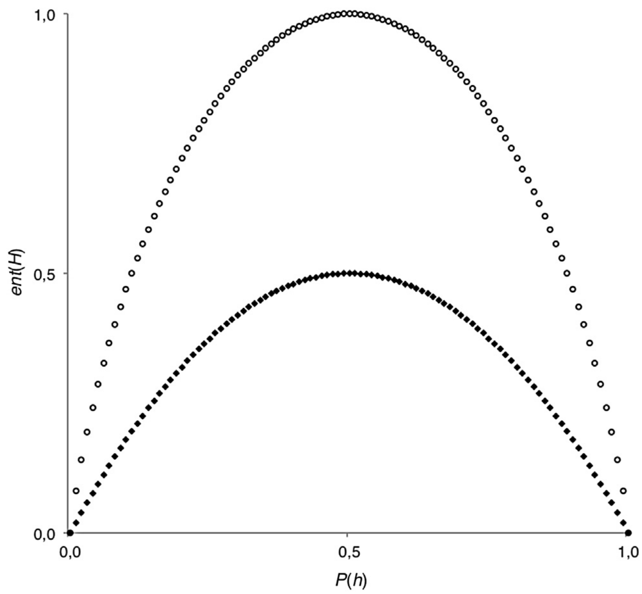

Fig. 3. A graphical illustration of ent_R (○) and ent_D (◆) as distinct measures of the entropy over a binary hypothesis set $H = \{h,\neg h\}$ as a function of the probability of h.

where $ent(H|e) = \sum_{h_i \in H} inf(h_i|e)P(h_i|e)$. The label information gain is sometimes taken as immediately denoting ig_R, a widely known model already presented by Bar-Hillel and Carnap (1953, p. 154), although under a rather idiosyncratic label (it would read "the amount of specification of H through e"). Nelson (2005) traces the ig_R measure back to Lindley (1956) and provides further references in cognitive science and beyond. As for ig_D, we found only a rather cursory occurrence in Niiniluoto and Tuomela (1973, p. 67). Indeed, the two models of information gain, ig_R and ig_D, are greatly different in popularity. However, they exhibit fairly similar quantitative patterns, as illustrated in Fig. 4 (again apart from issues of range), and both are additive as follows:

$$ig(H,e \wedge f) = ig(H,e) + ig(H,f|e)$$

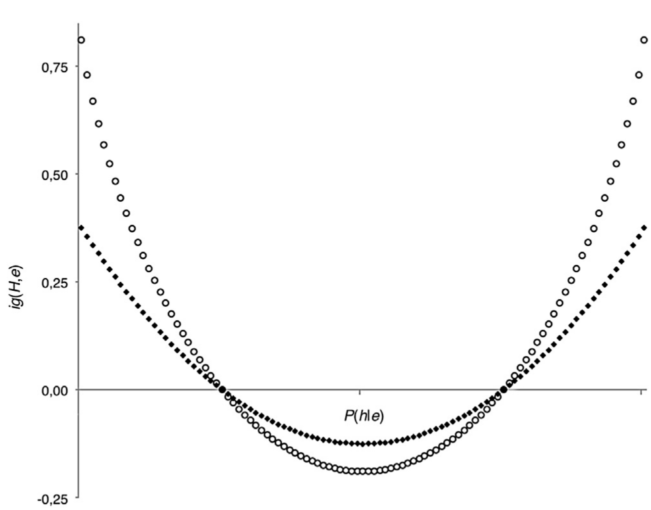

Fig. 4. A graphical illustration of ig_R (○) and ig_D (◆) as distinct measures of the information gain (entropy reduction) provided by evidence e concerning a binary hypothesis set $H = \{h,\neg h\}$ as a function of the posterior probability of h. Prior probability $P(h)$ is fixed at 0.25.

To illustrate, consider a clinical case and suppose that some diagnostic finding e raises the probability of the presence of some disease (h)—as simply contrasted to its absence ($\neg h$)—from 25% to 60%. Later on, a further comparably strong finding f drags the probability of h back down to 25%. Additive measures such as ig_R and ig_D see $ig(H,e \wedge f) = 0$ as arising from e having a negative informative value and f an equal and opposite (positive) value. Indeed, as said, given evidence can both reduce or increase the initial entropy concerning H, and the corresponding value of information gain would be positive or negative, accordingly. Informally, it is positive if the posterior distribution is more extreme than the prior, negative if the opposite obtains. Based on a discussion of various illustrative examples, Nelson (2008) characterizes the additivity property above as an appealing feature of ig_R, and the same considerations apply to ig_D, too.

3.2. *From information gain to expected confirmation*

If a model of information gain is available, ig_R or ig_D, which assigns values of informativeness to particular items of evidence or data relative to a target query of epistemic interest, the natural step forward is to turn it into a measure of the (*expected*) *utility of an experiment*, averaging over all of the possible outcomes of such experiment (question, test). The usefulness of experiment E relative to some target hypotheses set H (with $H,E \in \mathbf{Q}$ and $P \in \mathbf{P}$), is then computed by:

$$eu(H,E) = \sum_{e_j \in E} ig(H,e_j)P(e_j)$$

Note that, if ig is additive—as is the case for both ig_R and ig_D—then eu will be additive, too, in the sense that, for any $H,E,F \in \mathbf{Q}$ and any $P \in \mathbf{P}$, $eu(H,E*F) = eu(H,E) + eu(H,F|E)$, where the latter quantity is the expected usefulness of F across all possible outcomes of the E test, that is, $eu(H,F|E) = \sum_{e_j \in E} eu(H,F|e_j)P(e_j)$.

Now take a confirmation measure $conf(h,e)$ which amounts to the difference $inf(h) - inf(h|e)$—recall that this is how we arrived at bc_R and bc_D early on. Further, take $ent(H)$ as the expected value of inf over H—that is the way in which ent_R and ent_D were derived in the foregoing section. Then a remarkable fact obtains, namely, for any $H,E \in \mathbf{Q}$ and any $P \in \mathbf{P}$:

$$\begin{aligned} eu(H,E) &= \sum_{e_j \in E}[ent(H) - ent(H|e)]P(e_j) \\ &= ent(H) - \sum_{e_j \in E} ent(H|e_j)P(e_j) \\ &= \sum_{h_i \in H} inf(h_i)P(h_i) - \sum_{e_j \in E}\sum_{h_i \in H} inf(h_i|e_j)P(h_i|e_j)P(e_j) \\ &= \sum_{e_j \in E}\sum_{h_i \in H} inf(h_i)P(h_i|e_j)P(e_j) \\ &\quad - \sum_{e_j \in E}\sum_{h_i \in H} inf(h_i|e_j)P(h_i|e_j)P(e_j) \\ &= \sum_{e_j \in E}\sum_{h_i \in H} P(h_i|e_j)[inf(h_i) - inf(h_i|e_j)]P(e_j) \\ &= \sum_{e_j \in E}\sum_{h_i \in H} P(h_i|e_j)conf(h_i,e_j)P(e_j) \end{aligned}$$

Note that $\sum_{h_i \in H} P(h_i|e)conf(h_i,e)$, in turn, amounts to an extension of the $conf$ measure across a whole hypothesis set, H: quite simply, specific $conf$ values from e to each $h_i \in H$ are picked up and then aggregated, with posterior probabilities serving as weights.[10] A specific label seems justified for this quantity. Here, we will employ *ec* (for extended confirmation) and define ec_R and ec_D accordingly:

$$ec_R(H,e) = \sum_{h_i \in H} P(h_i|e) bc_R(h_i, e)$$

$$ec_D(H,e) = \sum_{h_i \in H} P(h_i|e) bc_D(h_i, e)$$

Now, given the foregoing derivation, the usefulness of E for H can be represented in two equivalent ways: as the expected value of information gain, $ig(H,e)$, or alternatively as the expected value of confirmation (appropriately extended to sets of hypotheses), $ec(H,e)$. Depending on the choice between inf_R and inf_D as a basis to measure information, we then have, respectively:

$$eu_R(H,E) = \sum_{e_j \in E} ig_R(H,e_j)P(e_j) = \sum_{e_j \in E} ec_R(H,e_j)P(e_j)$$

$$eu_D(H,E) = \sum_{e_j \in E} ig_D(H,e_j)P(e_j) = \sum_{e_j \in E} ec_D(H,e_j)P(e_j)$$

At this point, we are fitting several pieces together. Based on two simple alternative ways to represent information—inf_R and inf_D, respectively—we can generate two models of the expected utility of experiments (questions, tests)—eu_R and eu_D. Each of these can be seen as arising from a distinct account of entropy reduction (viz., either ig_R or ig_D), or, alternatively, as relying on one classical Bayesian measure of confirmation as belief change (viz., either bc_R or bc_D). One reason why the above equalities are of notice is that *ig* and *ec* models, while identical in their expectation over E (namely, in the assessment of a question, E, relative to hypothesis set H), are *not* pairwise equivalent when it comes to the piecemeal assessments of individual answers (e.g., $e_j \in E$). For instance, ig_R, but not ec_R, is additive, whereas ec_R, but not ig_R, is non-negative (see Nelson, 2008).

The equivalence of ig_R and ec_R in their expectation over E was noted before (see, e.g., Oaksford & Chater, 1996, p. 391). The quantity ec_R, in particular, is well understood and otherwise generally known as the Kullback–Leibler (KL) divergence of $P(\bullet|e)$ from $P(\bullet)$ (after Kullback & Leibler, 1951; also, *relative entropy*, see Cover & Thomas, 1991, Ch. 2). By contrast, the analog connection between ig_D and ec_D is a novel finding, to the best of our knowledge. In fact, while there is at least one philosophical argument which can be reconstructed as an application of eu_D (Horwich's derivation that gathering evidence for a binary hypothesis set always has a positive expected epistemic utility, in 1982, pp. 127–129), we have been unable to find any previous occurrence of ec_D in the study of rationality and cognition.

We already know that eu_R and eu_D are both additive, and eu_R is also known to be non-negative, because $ent_R(H) \geq \sum_{e_j \in E} ent_R(H|e_j)P(e_j)$ for any $H,E \in \boldsymbol{Q}$ and any $P \in \boldsymbol{P}$ (a statement sometimes called Shannon's inequality; see, e.g., van Rooij, 2009, p. 176; Rosenkrantz, 1977, pp. 15–16). The non-negativity of $eu(H,E)$ captures the fundamental idea that for an agent with an interest in H it is never rational to pay someone *not* to tell him what is the true answer to query E. Appropriately, eu_D is non-negative, too, as shown by the following (see Fig. 5 for an illustration).

$$\begin{aligned}
eu_D(H,E) &= \sum_{e_j \in E}\sum_{h_i \in H} P(h_i|e_j) bc_D(h_i,e_j)P(e_j) \\
&= \sum_{e_j \in E}\sum_{h_i \in H} [(P(h_i|e_j) - P(h_i)) bc_D(h_i,e_j)P(e_j) \\
&\quad + P(h_i) bc_D(h_i,e_j)P(e_j)] \\
&= \sum_{e_j \in E}\sum_{h_i \in H} bc_D(h_i,e_j)^2 P(e_j) + \sum_{h_i \in H}\sum_{e_j \in E} bc_D(h_i,e_j)P(e_j)P(h_i) \\
&= \sum_{e_j \in E}\sum_{h_i \in H} bc_D(h_i,e_j)^2 P(e_j) \geq 0
\end{aligned}$$

because, as we know (see 2.1 above), $\sum_{e_j \in E} bc_D(h_i, e_j)P(e_j) = 0$ for each $h_i \in H$.

Given that $bc_D(h,e)^2 = [P(h|e) - P(h)]^2$, the final step of the above derivation reveals that $eu_D(H,E)$ also amounts to the expected value of the (squared) Euclidean distance of $P(\bullet|e)$ from $P(\bullet)$ over the elements of E. As a consequence of this further connection, the special case $eu_D(H,H)$ computes the expectation of a widely known measure of the *inaccuracy* of P concerning H: the Brier score (Brier, 1950; also see Pettigrew, 2013; Selten, 1998). The counterpart result for eu_R is that $eu_R(H,H)$ equals the expectation of the logarithmic score, an alternative measure of inaccuracy (Good, 1952; also see Gneiting & Raftery, 2007; Kerridge, 1961). So the expected logarithmic vs. Brier score of probability assignment P for hypothesis set H equals the expected utility of an ideal (conclusive) experiment, H itself, under eu_R vs. eu_D, respectively.[11]

It is easy to show that, if $eu(H,E)$ is additive and non-negative, then the following holds:

> (A) *Additional search*. For any $H,E,F \in \boldsymbol{Q}$ and any $P \in \boldsymbol{P}$, $eu(H,E{*}F) \geq eu(H,E)$, with equality in case H and F are independent given E, i.e., $H \perp_P F|E$.

(A) is a veritable cornerstone for the ordinal (comparative) behavior of any plausible theory of the usefulness of experiments (see Goosens, 1976 for a seminal discussion). It implies, in particular, that $eu(H,E)$ always ranges between $eu(H, \mathcal{T})$, the utility of the "empty" query $\mathcal{T} = \{\top\}$, and $eu(H,H)$, the utility of an ideal (conclusive) experiment, H itself. Moreover, one has $eu(H,E) = eu(H,\mathcal{T})$ in case E is irrelevant (that is, if $H \perp_P E$), and $eu(H,E) = eu(H,H)$ in case E is at least as fine-grained as H (that is, if $E = H{*}K$ for some $K \in \boldsymbol{Q}$). All these fundamental properties are thus retained regardless, no matter if eu_R or eu_D is preferred. Given the enormous imbalance in popularity in favor of eu_R, it is perhaps of interest to note a couple of considerations that would favor eu_D instead. Both such considerations, it turns out, map neatly onto similar issues that have been discussed in Bayesian confirmation theory, but seem to have remained unnoticed in the literature on information search and the utility of experiments.

Positive evidential support is not necessarily commutative when it comes to degrees. Following our earlier example, if e = "the card drawn is hearts" and h = "the card drawn is red", then surely e

[10] We must credit Gudny Gudmundsdottir for having first advocated in personal communication this method to extend bc_D to a whole set of hypotheses H.

[11] We thank Guglielmo Mereu for discussion on this point, which prompted us to make the connection explicit. Meanwhile, we came across van Enk's (2014) very interesting paper, showing how candidate Bayesian confirmation measures can be derived directly from popular scoring rules. van Enk (2014) finds appealing features in a measure $conf(h,e)$ amounting to $ec_R(H,e)$ for $P(h|e) \geq P(h)$ and to $-ec_R(H,e)$ for $P(h|e) < P(h)$, where $H = \{h,\neg h\}$. So, van Enk's (2014) favorite measure of the positive (negative) evidential impact of e on h is effectively nothing else than (minus) the Kullback–Leibler divergence of $P(\bullet|e)$ from $P(\bullet)$ over the binary partition $\{h,\neg h\}$. Rather surprisingly, this connection remains unnoticed in van Enk's (2014) discussion.

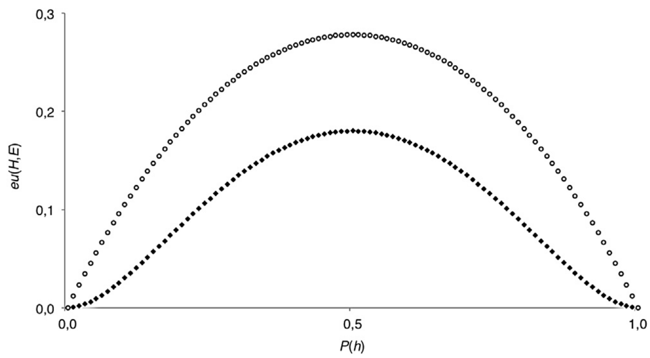

Fig. 5. A graphical illustration of eu_R (○) and eu_D (◆) as distinct measures of the usefulness of a binary test query $E = \{e,\neg e\}$ relative to binary target hypothesis set $H = \{h,\neg h\}$ as a function of the prior probability of h. Test characteristics are fixed by parameters $P(e|h) = P(\neg e|\neg h) = .80$.

and h support each other, but not with equal strength. Now suppose we adapt this example to a case where search for information, rather than given data, is at issue. A card was drawn from a well-shuffled standard deck and kept hidden. You're interested in the color of this card, $H = \{\text{red, black}\}$, and you can be told (truthfully) about its suit, $E = \{\text{hearts, diamonds, clubs, spades}\}$. How useful would this "experiment", E, be for your purposes? For comparison, suppose you want to know about the suit, and can get to know what the color is. What is the usefulness of the experiment allowed in this case (H) for the purposes that are now involved (knowing about E)? A strong intuition here is that $eu(H,E) > eu(E,H)$ should obtain, because E is a conclusive test for H, while H is not conclusive for E.[12] Although commutativity may be sound in engineering interpretations of the formalism (where the expectation of "transmitted information does not depend on the direction of flow", Rosenkrantz, 1977, p. 11), the mutual usefulness of queries H and E seems much closer to the case of positive support between single statements: here again, commutativity is all but compelling, when it comes to degrees. It is then a disturbing implication of eu_R that $eu_R(H,E) = eu_R(E,H)$ for any $H,E \in \mathbf{Q}$ and any $P \in \mathbf{P}$ (see, e.g., Bar-Hillel & Carnap, 1953, p. 156); $eu_D(H,E)$, on the contrary, yields the desired ranking in the card example, with $eu_D(H,E) = {}^1\!/_2 > {}^1\!/_4 = eu_D(E,H)$.

Commutativity bears additional inconveniences, too, for both confirmation and information search. Suppose H,E and K are queries (all binary, for simplicity) such that the latter is completely independent from the others (i.e., $H \perp_P K$, $E \perp_P K$, and $H{*}E \perp_P K$) and $e \in E$ and $h \in H$ are positively relevant to each other. By basic principle (F) of probabilistic confirmation, we have $conf(e,h) = conf(e,h \wedge k)$. But then, if commutativity holds, we concurrently have $conf(h,e) = conf(h \wedge k,e)$, and that's generally taken to troubling (in fact, a "paradox": see Crupi & Tentori, 2010; Fitelson, 2002). For degrees of confirmation from e to h should not be extended for free, as it were, to a more demanding (logically stronger) hypothesis $h \wedge k$, unless some appropriate confirmation relation holds between e and k itself. And that's another complaint concerning commutative confirmation measure bc_R. All this carries over neatly, it turns out, to information search. In fact, principle (A) implies that, if K is irrelevant as explained above, then $eu(E,H) = eu(E,H{*}K)$. If commutativity holds, moreover, $eu(H{*}K,E) = eu(H,E)$. Why is this bad? Well, suppose H and E are color and suit, once again, and K is the value of the card {2, 3, …, king, ace}. Then, intuitively, experiment E should not count as equally informative for $H{*}K$ as it is for H, because E is entirely *un*informative for K and moreover $H{*}K$ is a much more fine-grained hypothesis set than H, thus a much more demanding epistemic target. Indeed, E (suit) is again a conclusive test concerning H (color), but clearly not concerning $H{*}K$ (color *and* value). A commutative measure such as eu_R suffers from this problem of "irrelevant combination", too: $eu_R(H,E) = 1 = eu_R(H{*}K,E)$, in our current example. Measure eu_D, on the contrary, gives the ranking desired: $eu_D(H,E) = {}^1\!/_2 > 1/26 = eu_D(H{*}K,E)$.

4. Concluding remarks

Alternative theories of the expected utility of experiments or information search options raise issues that are similar to the plurality of confirmation measures (Brössel, 2013; Crupi et al., 2007; Festa, 1999; Fitelson, 1999). Importantly, this is not a matter of people having different credences or attitudes. Even for a single rational agent in a well-defined setting, the epistemic usefulness of two queries may be ranked differently by models such as eu_R and eu_D, thus implying contrasting criteria of optimality. As a consequence, much as for confirmation, treatments of information search will be sensitive to the choice of a particular model for measuring the usefulness of questions. In light of our discussion and other extensive analyses (see Nelson, 2005, 2008), it seems fair to say that the search for a firm theoretical basis for this choice is still underway. But the path which led us here provides some significant implications nonetheless, we think.

Consider for instance recent work purportedly showing that Bayesian confirmation theory is "a means with no end" (Brössel & Huber, 2014). The argument crucially relies on the claim that Bayesian confirmation theory is not used "to determine the epistemic value of experimental outcomes, and thus to decide which experiments to carry out". This complaint is amazing, judging by the tight connections between confirmation and information search that we have explored here and by a number of earlier suggestions from Bar-Hillel and Carnap (1953) to Good (1983), and beyond. Such issues might have remained latent in recent work on information search in human cognition (Austerweil & Griffiths, 2011; Crupi, Tentori, & Lombardi, 2009; Nelson, McKenzie, Cottrell, & Sejnowski, 2010; Oaksford & Chater, 2003), but even there important exceptions exist (see Fitelson & Hawthorne, 2010; Klayman & Ha, 1987; Nickerson, 1996; Rusconi, Marelli, D'Addario, Russo, & Cherubini, 2014).

Treating information and confirmation in a unified fashion is intuitive as well as desirable. Indeed, various existing models of optimal information search do essentially compute, one way or another, the utility of a question as a weighted average of the confirmatory impact of its possible answers. Once the connection is disclosed, moreover, it naturally brings to light potentially interesting theoretical options which might have escaped due attention. Consider, by way of illustration, the critical role that information search plays in clinical reasoning. Benish (1999) observed that "an appropriate mathematical model of diagnostic information has yet to be introduced" (p. 202) and went on to present eu_R as filling the gap (Benish, 2002, 2003), but failed to carry out any comprehensive comparison of the relative virtues of competing proposals, such as eu_D and others (see Card & Good, 1974; Good & Card, 1971; also see Nelson et al., 2013 for some steps towards such a comparison). More generally, it is not uncommon to find various fragments of this theoretical framework adopted (if not independently rediscovered) in specific settings or subdisciplines, without the benefit of an overview of the options and results already available (the legal domain offers further examples: see, e.g., Davis & Follette, 2002; Kaye & Koehler, 2003).

[12] By the way, we found this judgment to be largely shared in an informal sample of logicians, formal epistemologists, and psychologists of reasoning at a recent workshop (the yearly conference of the *New Frameworks of Rationality* project, Etelsen, 5–7 March 2013; see http://www.spp1516.de).

Providing basic tools to analyze the expected utility of information search options might well end up being one of the most fruitful applications of Bayesian confirmation theory. In our discussion, we did not review an extensive set of alternative confirmation measures. We rather tentatively accepted a distinction between two notions of confirmation or evidential support (belief change and partial entailment/refutation) and accordingly discussed two pairs of measures (bc_R vs. bc_D, and $part_R$ vs. $part_D$, respectively) naturally arising from two popular ways to represent the informative import of a statement (inf_R vs. inf_D). Although under this fairly selective approach, we showed that here, much as in the case of information search, the collection of models based on inf_D exhibits properties that are distinct and attractive as compared to those based on inf_R. In our view, this indicates that the differences between competing confirmation measures have theoretical meaning and thus partly counters skeptical or dismissive attitudes towards this debate (see, e.g., Howson, 2000, pp. 184–185; Kyburg & Teng, 2001, pp. 98 ff.).

The foregoing analysis could also be extended at least in two directions. First, our setting would allow for the inclusion of further models, particularly those hinging on likelihood and odds ratios (Good, 1950, 1983). Second, our discussion illustrates that the parallelism between confirmation and information search can serve as a valuable heuristic for theoretical work. A property like commutativity, for example, carries over from bc_R to eu_R, raising quite similar concerns in both cases. Considerations of this kind might inspire the development of new axiomatic treatments directly targeting models of the usefulness of experiments (rather than, in particular, the underlying entropy measures). A suggestive idea is, for instance, whether the commutativity property $eu(H,E) = eu(E,H)$ might be sufficient to single out eu_R (up to ordinal equivalence) if combined with the basic principle of additional search (A), or some variant of the latter. But such developments we have to postpone to some other occasion.

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