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A VEROSIMILITUDINARIAN ANALYSIS OF THE LINDA PARADOX

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ABSTRACT: The Linda paradox is a key topic in current debates on the rationality of human reasoning and its limitations. We present a novel analysis of this paradox, based on the notion of verisimilitude as studied in the philosophy of science. The comparison with an alternative analysis based on probabilistic confirmation suggests how to overcome some problems of our account by introducing an adequately defined notion of *verisimilitudinarian* confirmation.

Key words: Conjunction fallacy, Linda paradox, Verisimilitude, Truthlikeness, Probability, Confirmation, Verisimilitudinarian confirmation.

The notion of verisimilitude or truthlikeness of a scientific theory was introduced by Karl Popper, who claimed that the main epistemic goal of science is truth approximation and that scientific progress consists in devising new theories which are closer to the truth than preceding ones (Popper, 1963). It was then extensively explored in post-Popperian theories of verisimilitude (Niiniluoto, 1987; Kuipers, 2000; Oddie, 2008). In what follows, we further pursue this line of research, by showing how a *verisimilitudinarian* analysis can be fruitfully applied also to problems emerging at the interface between epistemology and the cognitive sciences. More precisely, we focus on the so called *Linda paradox*, a key topic in current debates on the rationality of human reasoning and its limitations. In section 1, we present the Linda paradox and briefly survey the different attempts, made by both psychologists and epistemologists, to provide a satisfactory account of this phenomenon. Then, in section 2, we propose a new account based on the notion of (expected) verisimilitude, which generalizes a previous attempt in the same direction (Cevolani et al., 2010). Finally, in section 3, we compare our account with an alternative one based on Bayesian confirmation theory (Crupi et al., 2008; Tentori et al., 2012) and define a notion of *verisimilitudinarian confirmation* allowing to overcome some of the limitations of the former.

1 The Linda paradox

In a seminal work on the psychology of reasoning and judgment under uncertainty, Amos Tversky and Daniel Kahneman presented the following description of a fictitious character, Linda, which would

then become famous (Tversky & Kahneman, 1983, 297):

> Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

In a series of experimental inquiries, Tversky and Kahneman asked several samples of participants (both statistically naïve and sophisticated subjects) to judge the probability of some hypotheses about Linda, including the isolated statement "Linda is a bank teller" (b from now on) and the conjunctive statement "Linda is a bank teller and is active in the feminist movement" ($b \wedge f$). The results showed a strong tendency to judge $b \wedge f$ as more probable than b. In a particularly neat demonstration of the phenomenon, 142 university students were simply asked to choose the more probable state of affairs between b and $b \wedge f$: 85% of them chose the latter. This pattern of judgments is puzzling in that it conflicts with a basic and uncontroversial principle of probability theory, known as the "conjunction rule", prescribing that a conjunction of statements can not be more probable than any of its conjuncts. This "Linda paradox" is only an instance of a widespread and well documented phenomenon, usually known in the literature as the "conjunction fallacy" or the "conjunction effect". Indeed, Tversky and Kahneman themselves, along with many others in subsequent investigations, were able to replicate this phenomenon in a variety of experimental scenarios, including real-life cases like, for instance, examples of medical prognosis (Tversky & Kahneman, 1983, 301).

In the last decades, scholars interested in the analysis of human reasoning and decision making under uncertainty devoted a great deal of attention to the Linda paradox (Hertwig & Gigerenzer, 1999; Samuels et al., 2002). However, the attempt of providing a satisfactory account of the phenomenon has proved rather challenging. The psychological literature has mainly focused on whether and how the effect is modulated by several variants of the experimental task (see, for example, (Wedell & Moro, 2008)). Interestingly, ever since Isaac Levi's insightful review of Kahneman, Slovic, and Tversky's influential work (1982), the Linda paradox has attracted the attention of a number of epistemology scholars (Levi, 1985; Bovens & Hartmann, 2003; Hintikka, 2004; Crupi et al., 2008; Peijnenburg et al., 2012). Their accounts can all be seen as attempts to flesh out the otherwise esoteric statement by Tversky and Kahneman themselves that "*feminist bank teller* is a better hypothesis about Linda than *bank teller*" (Tversky & Kahneman, 1983, 311). In what follows, we will explore a different strategy to account for this noteworthy remark by providing a verisimilitudinarian analysis of the problem. In a nutshell, we will show that "feminist bank teller", while less likely to be *true* than "bank teller", may well be more likely to be a better approximation to the *whole truth* about Linda.

2 A verisimilitudinarian account of the Linda paradox

2.1 Verisimilitude vs probability

In general terms, a statement, hypothesis or theory h is highly verisimilar if it says many things about the target domain, and if many of these things are (almost exactly) true. Thus, the (degree of) verisimilitude of h must depend on both its *content*, i.e., how much h says, and its *accuracy*, i.e., how much of what h says is in fact true. In Popper's words, verisimilitude "represents the idea of approaching comprehensive truth. It thus combines truth and content" (Popper, 1963, p. 237).

An important consequence of the fact that verisimilitude is a combination of truth and content — or a "mixture of truth and information" (Oddie, 1986, 12) — is that the notions of verisimilitude and probability have to be carefully distinguished. In particular, it is possible that h is expected to be quite close to the whole truth about the domain, but still h is not expected to be true, i.e., h is not probable. In fact, while (conditional) probability is a *decreasing* function of content, (expected) verisimilitude must be *positively* associated to high content. This is simply because "nothing is as close to the truth as the whole truth itself" (Oddie, 1986, 11), the latter clearly being a uniquely accurate *and exhaustive* description of a given matter of interest.[1]

This last remarks suggests how a verisimilitudinarian account of the Linda paradox can work. The basic idea is that experimental participants may judge $b \wedge f$ a better hypothesis about Linda as compared to b because they evaluate $b \wedge f$ as more verisimilar than b (Cevolani et al., 2010). In fact, the hypothesis "feminist bank teller", while less likely to be true than "bank teller", may well be evaluated as a better approximation to the whole truth about Linda. In this connection, it is perhaps surprising that — even without mentioning the idea of verisimilitude, of which they were probably unaware — Tversky and Kahneman themselves anticipated the basic intuition underlying a verisimilitudinarian analysis of the Linda paradox, as the following telling quotation reveals (Tversky & Kahneman, 1983, 312):

> The expected value of a message can sometimes be improved by increasing its content, although its probability is thereby reduced. [...] Consider the task of ranking possible answers to the question "What do you think Linda is up to these days?" The maxim of value could justify a preference for $b \wedge f$ over b in this task, because the added attribute *feminist* considerably enriches the description of Linda's current activities at an acceptable cost in probable truth.

[1] More precisely, if h logically entails g, but g does not entail h, then h is less probable, but may well be more verisimilar, than g. Indeed, if h and g are both true, and h is logically stronger than g, then h is more verisimilar than g according to most verisimilitude theorists (an exception being (Oddie, 1986)).

In other words, we suggest that experimental participants are assessing the expected verisimilitude, not the probability, of the two hypotheses involved in the Linda problem, and that this can explain their preference for $b \wedge f$ over b.

2.2 Expected verisimilitude of conjunctive hypotheses

To illustrate our analysis, we introduce a general framework to address the Linda paradox, based on the so-called basic feature approach to verisimilitude (Cevolani et al., 2011). This approach focuses on *conjunctive hypotheses*, i.e., hypotheses expressed as conjunctions of logically independent atomic statements of a language $\mathscr{L}$ describing a target domain $\mathscr{U}$. The (degree of) verisimilitude $Vs(h)$ of a conjunctive hypothesis h expresses the closeness or similarity of h to "the whole truth" about $\mathscr{U}$, construed as the most complete true description of the basic features of $\mathscr{U}$ in $\mathscr{L}$. Since, in most interesting cases, the whole truth is simply unknown, the estimated verisimilitude of competing hypotheses, not their actual verisimilitude, is the crucial point of interest. Accordingly, the (degree of) expected verisimilitude $EVs(h|e)$ of h on e expresses its estimated degree of closeness to the truth with respect to the available evidence e (Oddie, 1986; Niiniluoto, 1987). Given an epistemic probability distribution $p(\cdot)$ over the set of the relevant alternative states of affairs (possible worlds) $c_1, \ldots, c_n$ and a measure $Vs(h, c_i)$ of the closeness of h to a given state of affairs c_i, $EVs(h)$ is defined as follows:

$$EVs(h|e) = \sum_{c_i} Vs(h, c_i) \times P(c_i|e) \tag{1}$$

We suggest that, in order to account for the Linda paradox, one should focus on the expected verisimilitude of the two (conjunctive) hypotheses involved — i.e., on $EVs(b|e)$ and $EVs(b \wedge f|e)$, where e is Linda's description—, and on their relationship.

Verisimilitude theorists have proposed a number of (partially conflicting) measures Vs of verisimilitude. Most of these measures, however, agree with each other as far as the verisimilitude of a conjunctive hypotheses h is concerned. In particular, most of them are *monotonic* in the sense that the verisimilitude of h increases with the addition of true conjuncts, and decreases with the addition of false conjuncts (Cevolani et al., 2011, 187). As an example, a monotonic verisimilitude measure Vs would order the two hypotheses involved in the Linda scenario as follows:

$$\begin{aligned} &\text{if } f \text{ is true in } c_i, \text{ then} \quad Vs(b \wedge f, c_i) > Vs(b, c_i) \\ &\text{if } f \text{ is false in } c_i, \text{ then} \quad Vs(b \wedge f, c_i) < Vs(b, c_i) \end{aligned} \tag{2}$$

One can then prove that, under suitably defined conditions, for all monotonic verisimilitude measures

Vs:[2]

$$EVs(b \wedge f|e) > EVs(b|e) \text{ iff } P(f|e) > \sigma \tag{3}$$

where σ is a threshold value depending on the specific measure *Vs*. In words, if "Linda is a feminist" is sufficiently likely to be true given Linda's story (i.e., $P(f|e) > \sigma$), "feminist bank teller" is estimated as more verisimilar than "bank teller" alone. This makes the former a better hypothesis about Linda than the latter, thus explaining both people's preferences and Tversky and Kahneman's "esoteric" remark quoted in section 1.

3 Verisimilitude and confirmation

According to the analysis presented in the previous section, people tend to prefer "feminist bank teller" ($b \wedge f$) over "bank teller" alone (b) because they judge f sufficiently probable given Linda's story (e) and hence evaluate $b \wedge f$ as a better approximation to the whole truth about Linda than b.

As one can see from theorem 3, the relatively high probability of f given e is considered as the main factor underlying the Linda paradox. This is a point that the present analysis has in common with most alternative accounts of this phenomenon. However, this aspect of the verisimilitudinarian account is problematic, as the following case shows (Tentori et al., 2012). Suppose that a new item of information is added in the Linda scenario, i.e., the statement that "Linda owns an umbrella" (u). It seems clear that u is extremely likely to be true (since almost everybody owns an umbrella) and at least as probable as f, even on the background assumption that e. Accordingly, by the same pattern of reasoning described above, participants should prefer $b \wedge u$ over b alone as an hypothesis about Linda. In fact, one can see from theorem 3 that $b \wedge u$ is estimated as more verisimilar than b when $P(u) > \sigma$. In turn, since we are assuming that $P(u) \geq P(f)$, inequality $P(u) > \sigma$ holds whenever $P(f) > \sigma$. It follows that participants should prefer $b \wedge u$ to b whenever they prefer $b \wedge f$ to b.

The problem with this account is that, although f and u are both highly probable given e, only the former is *confirmed* by e, in the sense that its probability is increased by e. On the contrary, we can assume that the probability of u remains unchanged after learning e. Indeed, recent experimental investigations (Tentori et al., 2012) show that, when u is added in the Linda scenario, participants are much less prone to the conjunction fallacy; more precisely, the number of subjects preferring $b \wedge u$ to b is much lesser than the number of those judging $b \wedge f$ a better hypothesis than b about Linda.

A possible explanation of these results is that the key variable that generates the effect is not the high *posterior probability* of the added conjunct (like f or u), but the (*probabilistic*) *confirmation*

[2]Proofs are omitted through the paper; for details, see (Cevolani et al., 2012).

that the added conjunct receives from Linda's story. In general terms, the (degree of) confirmation $c(h,e)$ given by evidence e to hypothesis h expresses how much the probability of h increases (or decreases) once e is learned (Festa, 1999; Crupi & Tentori, 2012). As an example, the well-known "difference measure" of confirmation is defined as follows (Carnap, 1962, 361):

$$c(h,e) = p(h|e) - p(h) \tag{4}$$

Note that $c(h,e)$ is positive if and only if $P(h|e) > P(h)$, i.e., if the probability of h is increased by e. According to the "confirmation account" of the Linda paradox (Crupi et al., 2008), participants are not assessing the probability of the hypotheses at issue, but their confirmation given Linda's story. Under suitably defined conditions, this account can explain why participants prefer $b \wedge f$, but not $b \wedge u$, to b as an hypothesis about Linda. Indeed, this preference might be due to the fact that $c(b \wedge f, e) > c(b,e)$ while $c(b \wedge u, e) \leq c(b,e)$.

The intuitions underlying the confirmation account presented above and the verisimilitudinarian account of section 2 can be interestingly combined in what might be called the *verisimilitudinarian confirmation* account (*vs-confirmation* account for short) of the Linda paradox. The basic idea is that people might be assessing neither the verisimilitude, nor the confirmation, of the hypotheses involved in the Linda problem, but their vs-confirmation, which can be defined for instance as follows (cf. definition 4): given a verisimilitude measure *Vs*,

$$c_{Vs}(h,e) = EVs(h|e) - EVs(h) \tag{5}$$

Note that $c_{Vs}(h,e)$ is positive if and only if $EVs(h|e) > EVs(h)$; in other words, $c_{Vs}(h,e)$ expresses how much the expected verisimilitude of hypothesis h increases (or decreases) on the basis of evidence e.

According to the vs-confirmation account, "feminist bank teller" would be preferred to "bank teller" alone just in case $c_{Vs}(b \wedge f, e)$ were greater than $c_{Vs}(b,e)$, i.e., whenever $EVs(b \wedge f)$ is increased by e more than $EVs(b)$. One obtains a particularly interesting application of this account if definition 5 of vs-confirmation is phrased in terms of so called *additive* verisimilitude measures *Vs*. A measure *Vs* is additive when it is monotonic and, given a conjunctive hypothesis $h = h_1 \wedge \cdots \wedge h_k$ (where h_i are logically independent atomic statements), $Vs(h) = \sum_{h_i} Vs(h_i)$. Thus, for instance, if *Vs* is additive

then $Vs(b \wedge f) = Vs(b) + Vs(f)$.[3] One can then prove that, for all additive verisimilitude measures Vs,

$$c_{Vs}(b \wedge x, e) \gtreqless c_{Vs}(b, e) \text{ iff } c(x, e) \gtreqless 0 \tag{6}$$

i.e., that $b \wedge x$ is vs-confirmed by e more than b just in case the added conjunct x is (probabilistically) confirmed by e. An attractive feature of the vs-confirmation analysis of the Linda paradox is that it can easily accommodate the umbrella case. In fact, such case was originally introduced in order to suggest that f is confirmed by e while u is not, i.e., that the following inequalities hold: $c(f, e) > 0 \geq c(u, e)$. By theorem 6, this implies that $b \wedge f$, but not $b \wedge u$, is more vs-confirmed by e than b, i.e., that $c_{Vs}(b \wedge f, e) > c_{Vs}(b, e) \geq c_{Vs}(b \wedge u, e)$. In turn, this provides an explanation for the participants' preferences based on the notion of verisimilitudinarian confirmation.

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[3] A family of additive verisimilitude measure is provided by so called *feature contrast measures* of verisimilitude (Cevolani et al., 2011).

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